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A Sequence of Henstock Equi ϑ –integrable in A Nondicrete Locally Compact Metric Space

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Abstract Let X be a nondiscrete locally metric space, $\mathcal{E}(X)$ be a collection of elementary sets in X, and ϑ be a continuous volume function on $\mathcal{E}(X)$. In this paper, we will discuss that: if (g_n) is Henstock ϑ –integrable on a cell E in X and (g_n) is uniformly convergent to a function g on a cell E, then (g_n) has functionally small Riemann sum property with respect to ϑ uniformly in n on a cell E.

Key words: nondiscrete locally compact metric space, elementary set, cell.

1. Introduction

In 1995, Soeparna discussed some small Riemann sum properties in $\mathbb{R}$. In 2002, Manuharawati generalized some small Riemann sum properties in a nondiscrete locally compact metric space (X, d). The main goal this paper is to prove that a sequence has functionally small Riemann sum (FSRS) property with respect to ϑ uniformaly in n on a cell E if the sequence of function (g_n) is Henstock equi ϑ –integrable on a cell E and (g_n) is uniformly convergent to a function g on a cell E.

2. Basic Concept

Given a system of intervals $\mathcal{S}$ in nondiscrete locally metric space (X, d). A A collection of all elementary sets in X is denoted by $\mathcal{E}(X)$. If E is a compact elementary set with $int(E) \neq \emptyset$ and $\delta: E \to \mathbb{R}^+$ is a function, then a fine collection of cell-point pairs

$$\mathcal{P} = \{(A_{x_i}, x_i), 1 \leq i \leq n\} = \{(A_x, x)\}$$

Is called a Perron δ –fine partition on E if $x_i \in A_{x_i} \subset N(x_i, \delta_{x_i})$ and $\{A_{x_i}\}$ is partition on E.

Let ϑ be a continuous volume function on $\mathcal{E}(X)$, and $\mu^*: 2^X \to \mathbb{R}^+$ be an outer measure on X generated by ϑ. A function $g: X \to \mathbb{R}$ is said to be Henstock ϑ –integrable on a cell $E \subset X$, denoted by $g \in H(E, \vartheta)$ if there exists a real number

α with property: for any real number $\varepsilon > 0$ there exists a function $\delta: E \to \mathbb{R}^+$ such that for each Perron δ –fine partition

$$\mathcal{P} = \{(A_{x_i}, x_i), 1 \le i \le n\} = \{(A_x, x)\}$$

of E we have

$$\left|\sum_{i=1}^{n} g(x_i)\,\vartheta(A_{x_i}) - \alpha\right| = \left|\mathcal{P}\sum g(x)\,\vartheta(A_x) - \alpha\right| < \varepsilon.$$

3. Functionally Small Reimann Sum

Given a cell $E \subset \mathcal{S}$ and function $g: X \to \mathbb{R}^*$. A function g is said to have functionally small Reimann sum ($FSRS$) property with respect to ϑ on E, denoted by $g \in FSRS(E, \vartheta)$ if g is μ^* –measurable on E and for real number $\varepsilon > 0$ there exist a nonnegative μ –integrable function f on E and a function $\delta: E \to \mathbb{R}^+$ such that for any Perron δ –fine partition

$$\mathcal{P} = \{(A_x, x)\}$$

on E we have

$$\left|\mathcal{P}\sum_{|g(x)|>f(x)} g(x)\mu(A_x)\right| < \varepsilon.$$

Given a function $g_n: X \to \mathbb{R}^*$. A sequence of function (g_n) is said to have $FSRS$ property with respect to ϑ uniformly in n (**uniformly $\boldsymbol{\vartheta}$ – $\boldsymbol{FSRS}$ in $\boldsymbol{n}$**) on a cell $E \subset X$ if for each n, the function g_n is μ^* –measurable on E and for real number $\varepsilon > 0$ there exist a nonnegative μ –integrable function f on E and a function $\delta: E \to \mathbb{R}^+$ which is independent on n such that for any Perron δ –fine partition

$$\mathcal{P} = \{(A_x, x)\}$$

of E and for each n we have

$$\left|\mathcal{P}\sum_{|g_n(x)|>f(x)} g(x)\mu(A_x)\right| < \varepsilon.$$

A sequence of function (g_n) is said to be Henstock equi integrable with respect to ϑ (Hentock equi ϑ integrable) on a cell $E \subset X$ if for real number $\varepsilon > 0$ there exist a nonnegative μ –integrable function f on E and a function $\delta: E \to \mathbb{R}^+$ which is independent on n such that for any Perron δ –fine partition

$$\mathcal{P} = \{(A_x, x)\}$$

of E and for each n we have

$$\left|(H)\int_E g_n\, d\vartheta - \mathcal{P}\sum g_n(x)\vartheta(A_x)\right| < \varepsilon.$$

Some main properties of functionally small Reimann sum properties are discussed as fololows.

Theorem 3.1 (Manuharawati, 2002: 141): *Given a cell $E \subset X$ and functions $g, g_n: X \to \mathbb{R}^*$. If a sequence of a functions (g_n) is uniformly $\vartheta -FSRS$ in n on E and converges to a function g almost every where on E, then $g \in FSRS(E, \vartheta)$.*

Lemma 3.2 (Manuharawati 2006): *Given a cell $E \subset X$ and functions $g, g_n: X \to \mathbb{R}^*$. If a sequence of a functions (g_n) is Henstock equi $\vartheta -$ integrable on E and converges to a function g almost every where on E, then $g \in H(E, \vartheta)$.*

Theorem 3.3 *Given a cell $E \subset X$ and functions $g, g_n: X \to \mathbb{R}^*$. If a sequence of a functions (g_n) is Henstock equi $\vartheta -$ integrable on E and (g_n) is uniformly convergent to a function g on E, then (g_n) has FSRS property with respect to ϑ uniformly in n on a cell E.*

Proof. Since the (g_n) is Henstock equi $\vartheta -$ integrable on E and (g_n) is uniformly convergent to a function g on E, then by Lemma 3.2, $g \in H(E, \vartheta)$. So, applying the Theorem 3.1, $g \in FSRS(E, \vartheta)$. Let a real number $\varepsilon > 0$ be given. Since $g \in FSRS(E, \vartheta)$, then there are a nonnegative function $f \in L(E, \vartheta)$ and $\delta_0: E \to \mathbb{R}^+$ such that for every Perron δ_0 −fine $\mathcal{P}_0 = \{(A_x, x)\}$ on E, we have

$$\left| \mathcal{P}_0 \sum_{|g(x)|>f(x)} g(x)\mu(A_x) \right| < \frac{\varepsilon}{3}.$$

.Since the sequence (g_n) is Henstock equi $\vartheta -$ integrable on E, then for every n, $g_n \in H(E, \vartheta)$. By Theorem 3.1, $g \in FSRS(E, \vartheta)$, So for a real number $\varepsilon > 0$ in above, there are a nonnegative function $f_n \in L(E, \vartheta)$ and $\delta_i: E \to \mathbb{R}^+$ such that for every Perron δ_n −fine $\mathcal{P}_n = \{(A_x, x)\}$ on E, we have

$$\left| \mathcal{P}_n \sum_{|g_n(x)|>f(x)} g_n(x)\mu(A_x) \right| < \frac{\varepsilon}{3}.$$

Since (g_n) is uniformly convergent to a function g on E, then there is a natural number N such that for every $n \in \mathbb{N}, n \geq N$ and for every $x \in E$ we have

$$\left| g_n(x) - g(x) < \frac{\varepsilon}{3(\vartheta(E) + 1)} \right|.$$

Define a function

$$\delta(x) = \min\{\delta_0(x), \delta_n(x): 1 \leq n \leq N - 1\}.$$

If we choose a function $h: E \to \mathbb{R}^+$ with

$$h(x) = \max\{f(x), f_n(x): 1 \leq n \leq N - 1\}$$

then a function h is nonnegative and $h \in L(E, \mu)$. So, for any Perron δ −fine partition $\mathcal{P} = \{(A_x, x)\}$ on E we have:

(i) For $n \leq N - 1$,

$$\left|\mathcal{P}\sum_{|g_n(x)|>h(x)} g_n(x)\mu(A_x)\right| < \left|\mathcal{P}\sum_{|g_n(x)|>f_n(x)} g_n(x)\mu(A_x)\right|\frac{\varepsilon}{3}.$$

(ii) For $n \geq N$

$$\left|\mathcal{P}\sum_{|g_n(x)|>h(x)} g_n(x)\mu(A_x)\right|$$

$$\leq \left|\mathcal{P}\sum_{|g_n(x)|>h(x)} [g_n(x) - g(x)]\,\mu(A_x)\right|$$

$$+ \left|\sum_{|g_n(x)|>h(x)} g(x)\mu(A_x)\right| < \varepsilon.$$

It means that a sequence (g_n) has FSRS property with respet to ϑ uniformly in n on acell E.

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